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Modelling of Convective Heat and Mass Transfer in Nanofluids With and Without Boiling and Condensation



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Preface

This monograph focuses on the methodology and results of the analytical and numerical modeling of convective heat and mass transfer in nanofluids and ordinary fluids subject to different effects. The book introduces the main points of the symmetry group analysis and self-similar forms for laminar and turbulent flows of nanofluids, as well as to perturbation method to study flow instabilities of different nature. The main part of the book is devoted to analytical modeling of boundary layers of nanofluids, jet impingent onto an orthogonal wall, film condensation of still and moving vapor with nanoparticles, stable film boiling of nanofluids, instantaneous unsteady boiling and condensation of nano- and ordinary fluids, convective instability of the vapor layer, as well as centrifugal and Dean instability in nanofluids. It is completed with summary and conclusions. It was demonstrated that such complex phenomena can be successfully simulated using the self-similar and analytical solutions, as well as the perturbation method validated via reliable experiments.

There are several books devoted to heat transfer in nanofluids that deal with many aspects of the physics and modeling of nanofluid flows. The book of D. Y. Shang and L. C. Zhong Heat Transfer Due to Laminar Natural Convection of Nanofluids. Theory and Calculation (Springer International Publishing, 2019) represents a theoretical study of heat transfer due to laminar natural convection of nanofluids. The authors used known similarity transformations as a modeling approach. In addition, the authors developed predictive relations for the prediction of heat transfer during natural convection of Al₂O₃-water nanofluids. The book of D. D. Ganji and A. Malvandi Heat Transfer Enhancement Using Nanofluid Flow in Microchannels. Simulation of Heat and Mass Transfer (Elsevier Inc., 2016) focuses on numerical simulations of heat transfer enhancement of nanofluids in microchannels. The book of E. E. S. Michaelides Nanofluidics. Thermodynamic and Transport Properties (Springer International Publishing, 2014) elucidates fundamentals of the transport processes using particle-fluid suspensions and overviews experimental, analytical, and numerical advances of different researchers. The author discusses also promising applications and technological issues of nanofluids.

However, the theoretical analysis and modeling approaches provided in the abovementioned books are often insufficient, whereas the authors used in the modeling the similarity functions and variables developed earlier and not considering the peculiarities of nanofluids. No analytical solutions have been derived. No model of turbulent flow with nanoparticles was developed. The issues of convective and centrifugal instability also have not been elucidated.

All the above has become a motivation for us to summarize in the form of a monograph; the results of our research are performed over the past more than ten years and published in a number of articles. The analysis, comparisons, and generalization of these results are performed here at the modern level with the involvement of the newly published theoretical and experimental findings of various authors. It was shown for the first time in a book format, how the Lie group analysis can be used to derive self-similar forms for parabolic single-phase flows and for film boiling of nanoliquids with an arbitrary dependence of the properties of the medium (viscosity, thermal conductivity, etc.) on the concentration of nanoparticles and temperature. The book summarizes the analytical solutions for two-phase flows of nanomedia during film condensation and boiling, the research results for instability of film boiling in the presence of nanoparticles, and centrifugal instability of nanofluid flows. The book also outlines criteria for both types of instability. Applications of liquids with nanoparticles in quenching processes were analyzed.

The present book consists of nine chapters. The book focuses on convective heat and mass transfer in boundary layers, during steady-state and unsteady condensation and boiling of nanofluids, as well as on the issues of instability at boiling and centrifugal instability. Chapters 1 and 2 are devoted to the physical foundations and methods of mathematical modeling of nanofluids. Chapters 3–8 present the results of solving various problems related to the flows of nanofluids, whereas each of them begins with an introduction, which provides a brief overview of the work of various researchers and formulates the purpose and objectives of this chapter.

Chapter 1 outlines physical foundations and mathematical models of transport processes in nanofluids. This chapter briefly shows that the addition of high thermal conductivity nanoparticles to conventional fluids intensifies heat transfer in them. Chapter 1 shows how mathematical models can consider the effect of nanoparticles on thermal conductivity, dynamic viscosity, heat capacity, and density of nanofluids. Later in this chapter, the thermophoresis and Brownian diffusion models are considered as the main physical mechanisms that influence the transport processes in nanofluids. In conclusion, a mathematical model of fluid flow and heat and mass transfer in nanofluids, which is used throughout this book, is presented in detail.

In Chap. 2, the foundations of two analytical mathematical methods used in mathematical modeling in this monograph are outlined: Lie group theory (symmetry analysis) and perturbation method.

Chapter 3 is devoted to an analysis of boundary layer flows (parabolic flows) of nanofluids as homogeneous single-phase fluids (i.e., flows without phase transition). This chapter outlines symmetry and self-similar forms of equations for different kinds of boundary layers on a flat surface. After that, self-similar solutions of the problems of flow and heat and mass transfer are presented for the laminar and turbulent boundary layers in the case of forced flow over a horizontal surface. In the

last section, the problem of orthogonal impingement of a flow onto a flat wall is considered.

In Chap. 4, results are presented for the problems of film condensation of (a) a stationary vapor with nanoparticles on a vertical surface and (b) a moving vapor that moves in the same direction with a film of condensate on a horizontal surface. The obtained analytical solutions are an extension and further elaboration of the classical Nusselt solution.

Chapter 5 describes the results for laminar flow, heat, and mass transfer in a vapor film on a vertical surface with developed steady-state film boiling of a stationary nanofluid. The model used here is a further development of the works of Bromley, Ellion, and Koh for ordinary fluids. As a result, an analytical solution to the problem was obtained. In conclusion, symmetry analysis was performed, and self-similar forms of transport equations were obtained, which were solved numerically. This solution was analyzed in comparison with the analytical solution.

In Chap. 6, results for the problem of instantaneous transition to film boiling in ordinary fluids and nanofluids on a vertical surface for the case of a sudden heat supply are outlined and analyzed. Here two approaches are used: the Laplace transform and the symmetry method. Self-similar equations obtained using the Lie group transformations were solved numerically. Validations were performed using the CFD methodology and experimental measurements of surface heat transfer of a metal probe, very quickly immersed in a cooling nanofluid.

Chapter 7 focuses on the study stability of the flow in a vapor film formed at boiling of nanofluids on a vertical surface, whereas the vapor flow in the film is directed vertically upward. The modeling was performed using the method of linear perturbations in two-dimensional and three-dimensional approximations. Experimental validation was carried out on the basis of data on the effect of nanoparticles on the formation and destruction of a vapor film formed during the boiling of nanofluid on the surface of a metal probe during unsteady cooling.

In Chap. 8, original results are presented of modeling using the perturbation method of the stability criteria in the case of instability during boiling, as well as centrifugal instability in the Dean and Taylor–Couette flows of nanofluids with account for the radial temperature gradient.

Chapter 9 presents summary and overall conclusions to the book.

The results that form the basis of this book were obtained during the last decade of our joint creative collaboration. We want to thank all of our colleagues with whom we have collaborated during this time, for their contributions, helpful advice, and friendly discussions.

We are very grateful to our families for their invaluable continued support and understanding in the preparation of this book.

Kyiv, Ukraine Gummersbach, Germany Andriy A. Avramenko Igor V. Shevchuk

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Nomenclature

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$Ar = \frac{g\delta^3}{\mu^2} \rho \Delta \rho$	Archimedes number (–)
$c_w = \frac{2\tau_w}{\rho_\infty U_\infty^2}$	Skin friction coefficient (–)
$c_{w0} = \frac{2\tau_{w0}}{\rho_{\infty}U_{\infty}^2}$	Skin friction coefficient for a pure base fluid (-)
c	Isobaric specific heat of a fluid (J/(kg K))
c_v	Isobaric specific heat of pure vapor without nanoparticles
	(J/(kg K))
<i>c</i> _p	Specific heat of the nanoparticles (J/(kg K))
D_B	Brownian diffusion coefficient (m ² /s)
D_{Bt}	Turbulent Brownian diffusion coefficient (m ² /s)
D_{Beff}	Effective Brownian diffusion coefficient (a sum of
	molecular and turbulent values) (m^2/s)
D_T	Thermophoretic diffusion coefficient (m ² /s)
D_{Bt}	Turbulent thermophoretic diffusion coefficient (m ² /s)
D_{Teff}	Effective thermophoretic diffusion coefficient (a sum of
	molecular and turbulent values) (m ² /s)
$\overline{D} = \frac{D_T}{D_B}$	Ratio of the thermophoretic diffusion coefficient to the
b	Brownian diffusion coefficient (–)
$D = \frac{\Delta T}{T_{\infty}} \frac{D_T}{D_B}$	Parameter (–)
d_p \sim b	Particle diameter (m)
$\operatorname{Ec} = \frac{U_{\infty}^2}{c_f \Delta T}$	Eckert number (–)
$Fo = \frac{\alpha t}{R^2}$	Fourier number based on the thermal diffusivity (–)
$Fo^* = \frac{tv}{\delta^2}$	Fourier number based on kinematic viscosity (-)
g	Acceleration of gravity (m/s^2)
G	Mass flowrate per unit length $(kg/(m \cdot s))$
G_l	Mass flow rate through the film per unit of surface area
	(kg/(m ² ·s))
$Ga = \frac{gx^3\rho_f^2}{\mu^2}$ $h = cT$	Galilei number (-)
h = cT	Specific enthalpy (J/kg)
h_w	Specific enthalpy of at the wall (J/kg)

h_∞	Specific enthalpy of the outer flow, i.e., the flow outside f the boundary lower $(1/2\pi)$
I.	of the boundary layer (J/kg)
h_T	Heat transfer coefficient $(W/(m^2 K))$
j_p	Mass flux of nanoparticles $(kg/m^2 \cdot s)$
$Ja_h = \frac{h_\infty}{L_v}$	Jacoby number (–)
k	Thermal conductivity of the nanofluid (W/(m K))
k_f	Thermal conductivity of the pure base fluid (W/(m K))
k_p	Thermal conductivity of the nanoparticles (W/(m K))
k_t	Turbulent thermal conductivity of the nanofluid (W/(m
	K))
$k_{e\!f\!f}$	Effective thermal conductivity of the nanofluid (a sum of
	molecular and turbulent values) (W/(m K))
$K_{pf} = k_p / k_f$	Normalized thermal conductivity of nanoparticles relative
	to that of the nanofluid (–)
$K_{pv} = k_p / k_v$	Normalized thermal conductivity of nanoparticles; rela-
1 1	tive to that of pure vapor (–)
$k_B = 1.38065 \cdot 10^{-23}$	Boltzmann constant $(J K^{-1})$
L_c	Characteristic length (m)
L_v	Latent heat of vaporization $(J kg^{-1})$
$Le = \alpha_f / D_B$	Lewis number (–)
$Le = \frac{k_f}{\rho_f c_f D_B} \frac{\rho_f c_f}{\rho_p c_p}$	Modified Lewis number of a pure base fluid based on the
$\rho_f c_f D_B \rho_p c_p$	Brownian diffusion coefficient (–)
$Le = \frac{Sc}{Pr} \frac{\rho_{\nu} c_{\nu}}{\rho_{n} c_{n}}$	Modified Lewis number of a pure vapor based on the
$\mathbf{L}\mathbf{c} = \Pr \rho_p c_p$	Brownian diffusion coefficient (–)
$N_{\rm H} = h_{\rm e} I_{\rm e} / h_{\rm e}$	Nusselt number for ordinary fluids (–)
$Nu = h_T L_c / k$ $Nu = h_T x$	-
$Nu = \frac{h_T x}{k_f}$	Local Nusselt number for nanofluids (–)
$Nu = \frac{h_T \delta 0}{k_f}$	Nusselt number for nanofluids based on the condensate
	film thickness (–)
$Pr = \frac{\mu_f c_f}{k_f} = v_f / a_f$ $Pr = \frac{\mu_v c_v}{k_v} = v_v / a_v$	Prandtl number of a pure base nanofluid (–)
$Pr = \frac{\mu_v c_v}{k} = v_v / a_v$	Prandtl number of a pure vapor (–)
p p	Static pressure (Pa)
Q	Heat flux (W)
q	Heat flux per unit area (W/m ²)
$q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0}$	Wall heat flux per unit area (W/m ²)
q_{cr}	Critical heat flux per unit area (W/m ²)
$R_{pf} = \rho_p / \rho_f$	Ratio of the densities of nanoparticles and pure base
рј гртгј	fluid (–)
$R_{\rm mu} = \frac{\rho_p}{2}$	Ratio of the densities of nanoparticles and pure vapor (–)
$R_{pv} = \frac{\rho_p}{\rho_v}$ Re = VL _c /v	Reynolds number (–)
$\operatorname{Re}_{x} = \frac{\rho_{f} U_{\infty} x}{\mu_{f}}$	Local Reynolds number for nanofluids (–)
$\mathbf{R}_{x} = \frac{\mu_{f}}{U_{\infty}\delta\rho}$	
$\operatorname{Re}_{\delta} = \frac{U_{\infty}\delta\rho}{\mu}$	Characteristic Reynolds number (-)
$\mathrm{Re}_{ au} = rac{\sqrt{ au_{\infty}/ ho}\delta}{\mu}$	Characteristic Reynolds number (-)
r	

Nomenclature

$\mathbf{S}_{2} \mu_{f} \nu_{f}$	Dorticle Schwidt number of a nonofluid based on the
$\mathrm{Sc} = rac{\mu_f}{ ho_f D_B} = rac{ u_f}{D_B}$	Particle Schmidt number of a nanofluid based on the
$\mathbf{S}_{\mathbf{a}} = \mu_{f} + \mu_{f}$	Brownian diffusion coefficient (–) Particle Schmidt number of a nanofluid based on the
$\mathrm{Sc}_T = rac{\mu_f}{ ho_f D_T} = v_f / D_T$	
$\mathbf{C}_{n} = \mu_{n} \dots D$	thermophoretic diffusion coefficient (–)
$\mathrm{Sc} = \frac{\mu_v}{\rho_v D_B} = v_v / D_B$	Particle Schmidt number of a pure vapor based on the
\mathbf{c}_{\star} h_{τ_0}	Brownian diffusion coefficient (–)
$\mathrm{St}_0 = rac{h_{T0}}{c_\infty ho_\infty U_\infty}$	Stanton number
$\widetilde{t} = \frac{t\nu}{\delta^2}$	Time (s)
$I = \frac{1}{\delta^2}$	Dimensionless time (-)
$t^* = \frac{\tilde{t}}{\frac{\tilde{r}}{Pr}}$	Dimensionless time (–)
	Local temperature (K)
T_w	Wall temperature (K)
T_{∞}	Ambient temperature, i.e., temperature outside of the
<i>T</i>	boundary layer (K)
T _{sat}	Saturation temperature at a given pressure (K)
V	Characteristic velocity (m/s)
v V	Fluid velocity vector (m/s)
$\nabla \mathbf{v}$	Tensor gradient of velocity $(s^{-1}m^{-1})$
$(\nabla \mathbf{v})^T$	Conjugate tensor gradient of velocity $(s^{-1}m^{-1})$
<i>v</i>	Wall-normal velocity component (y-component) (m/s)
<i>v</i> , <i>u</i> , <i>w</i>	Radial, azimuthal, and axial components of the flow value it is guindrical poler accordinates r_{i} (r_{i} (r_{i}))
77 N N	velocity in cylindrical polar coordinates r , φ , z (m/s) Velocity components in Cartesian coordinates (m/s)
<i>u</i> , <i>v</i> , <i>w</i>	• •
$u U_{\infty}$	Streamwise velocity component (<i>x</i> -component) (m/s) Outer flow velocity, i.e., flow velocity outside of the
U_{∞}	boundary layer (m/s)
<i>x</i> , <i>y</i> , <i>z</i>	Cartesian coordinates (m)
<i>λ</i> , <i>y</i> , <i>ζ</i> . Q	Volumetric thermal expansion coefficient of the nanofluid
ŭ	(K^{-1})
α	Thermal diffusivity of the nanofluid (m^2/s)
δ	Thickness of a momentum boundary layer, condensate
	film or vapor film (m)
$\Delta T = T_w - T_\infty$	Temperature difference in the boundary layer (K)
$\Delta T = T_w - T_{sat}$	Wall superheat during boiling (K)
$\eta = \frac{y}{\delta}$	Dimensionless coordinate in the condensate or vapor
0	film (–)
$\Theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}$ $\Theta = \frac{T - T_{\infty}}{T_w - T_{\infty}} \Theta = \frac{T_w - T}{T_w - T_{\infty}}$	Non-dimensional temperature (-)
$\Theta = \frac{T_w}{T - T_\infty} \Theta = \frac{T_w - T}{T - T_\infty}$	Non-dimensional temperature in the unsteady heat
$T_w - T_\infty$ $T_w - T_\infty$	transfer problems (–)
μ	Dynamic viscosity of the nanofluid (Pa s)
μ_f	Dynamic viscosity of the pure base fluid (Pa s)
μ_t	Turbulent viscosity of the nanofluid (Pa s)
μ_{eff}	Effective dynamic viscosity of the nanofluid (a sum of
	molecular and turbulent values) (Pa s)

ν	Kinematic viscosity of the nanofluid (m ² /s)
ρ	Density of the nanofluid (kg/m ³)
$ ho_f$	Density of the pure fluid (kg/m ³)
ρ_p	Density of the nanoparticles (kg/m ³)
ρ_{∞}	Density of the nanofluid outside of the boundary layer (kg/m^3)
ρ_v	Density of pure vapor (kg/m ³)
$\frac{\rho_{\nu}}{\bar{\rho}} = \rho_{\infty} / \rho_f$	Non-dimensional density outside of the boundary
	layer (–)
τ	Shear stress (Pa)
$ \tau_w = \mu_w \left(\frac{\partial u}{\partial y}\right)_{y=0} $	Shear stress on the wall (Pa)
φ	Volume fraction (concentration) of the nanoparticles (–)
φ_w	Volume fraction (concentration) of the nanoparticles, on
	the wall (–)
ϕ_{∞}	Volume fraction (concentration) of the nanoparticles, outside of the boundary layer (–)

Subscripts

- av Average value
- *f* Properties of a pure base fluid
- *p* Particles (i.e., nanoparticles)
- t Turbulent parameters
- v Properties of a pure vapor
- w Wall value (at y = 0)
- 0 Standard conditions: a pure base fluid
- ∞ Outer flow (i.e., outside of the boundary layer)

Mathematical Symbols

Boldface

 $\nabla^{2} = \left(\frac{\partial^{2}}{\partial r^{2}} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^{2}}\frac{\partial^{2}}{\partial \varphi^{2}} + \frac{\partial^{2}}{\partial z^{2}}\right)$ δ

Vector values Operator Nabla Operator Nabla quadrat Unit tensor (Kronecker tensor)

Acronyms

CFD	Computational fluid dynamics
DPHC	Direct problems of heat conduction
IPHC	Inverse problems of heat conduction
MATHLAB	Programming and numeric computing platform
VOF	Volume of liquid