Mathematical Engineering

Today, the development of high-tech systems is unthinkable without mathematical modeling and analysis of system behavior. As such, many fields in the modern engineering sciences (e.g. control engineering, communications engineering, mechanical engineering, and robotics) call for sophisticated mathematical methods in order to solve the tasks at hand.

The series Mathematical Engineering presents new or heretofore little-known methods to support engineers in finding suitable answers to their questions, presenting those methods in such manner as to make them ideally comprehensible and applicable in practice.

Therefore, the primary focus is—without neglecting mathematical accuracy—on comprehensibility and real-world applicability.

To submit a proposal or request further information, please use the PDF Proposal Form or contact directly: Dr. Thomas Ditzinger (thomas.ditzinger@springer.com)

Indexed by SCOPUS, zbMATH, SCImago.

More information about this series at https://link.springer.com/bookseries/8445

Andriy A. Avramenko · Igor V. Shevchuk

Modelling of Convective Heat and Mass Transfer in Nanofluids With and Without Boiling and Condensation

Springer

Andriy A. Avramenko
Institute of Engineering Thermophysics
National Acadamy of Sciences of Ukraine
Kyiv, Ukraine

Igor V. Shevchuk
Faculty of Computer Science
and Engineering Science
TH Köln/University of Applied Sciences
Gummersbach, Germany

ISSN 2192-4732 ISSN 2192-4740 (electronic)
Mathematical Engineering
ISBN 978-3-030-95080-4 ISBN 978-3-030-95081-1 (eBook)
https://doi.org/10.1007/978-3-030-95081-1

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This monograph focuses on the methodology and results of the analytical and numerical modeling of convective heat and mass transfer in nanofluids and ordinary fluids subject to different effects. The book introduces the main points of the symmetry group analysis and self-similar forms for laminar and turbulent flows of nanofluids, as well as to perturbation method to study flow instabilities of different nature. The main part of the book is devoted to analytical modeling of boundary layers of nanofluids, jet impingent onto an orthogonal wall, film condensation of still and moving vapor with nanoparticles, stable film boiling of nanofluids, instantaneous unsteady boiling and condensation of nano- and ordinary fluids, convective instability of the vapor layer, as well as centrifugal and Dean instability in nanofluids. It is completed with summary and conclusions. It was demonstrated that such complex phenomena can be successfully simulated using the self-similar and analytical solutions, as well as the perturbation method validated via reliable experiments.

There are several books devoted to heat transfer in nanofluids that deal with many aspects of the physics and modeling of nanofluid flows. The book of D. Y. Shang and L. C. Zhong *Heat Transfer Due to Laminar Natural Convection of Nanofluids. Theory and Calculation* (Springer International Publishing, 2019) represents a theoretical study of heat transfer due to laminar natural convection of nanofluids. The authors used known similarity transformations as a modeling approach. In addition, the authors developed predictive relations for the prediction of heat transfer during natural convection of Al_2O_3-water nanofluids. The book of D. D. Ganji and A. Malvandi *Heat Transfer Enhancement Using Nanofluid Flow in Microchannels. Simulation of Heat and Mass Transfer* (Elsevier Inc., 2016) focuses on numerical simulations of heat transfer enhancement of nanofluids in microchannels. The book of E. E. S. Michaelides *Nanofluidics. Thermodynamic and Transport Properties* (Springer International Publishing, 2014) elucidates fundamentals of the transport processes using particle-fluid suspensions and overviews experimental, analytical, and numerical advances of different researchers. The author discusses also promising applications and technological issues of nanofluids.

However, the theoretical analysis and modeling approaches provided in the above-mentioned books are often insufficient, whereas the authors used in the modeling the similarity functions and variables developed earlier and not considering the peculiarities of nanofluids. No analytical solutions have been derived. No model of turbulent flow with nanoparticles was developed. The issues of convective and centrifugal instability also have not been elucidated.

All the above has become a motivation for us to summarize in the form of a monograph; the results of our research are performed over the past more than ten years and published in a number of articles. The analysis, comparisons, and generalization of these results are performed here at the modern level with the involvement of the newly published theoretical and experimental findings of various authors. It was shown for the first time in a book format, how the Lie group analysis can be used to derive self-similar forms for parabolic single-phase flows and for film boiling of nanoliquids with an arbitrary dependence of the properties of the medium (viscosity, thermal conductivity, etc.) on the concentration of nanoparticles and temperature. The book summarizes the analytical solutions for two-phase flows of nanomedia during film condensation and boiling, the research results for instability of film boiling in the presence of nanoparticles, and centrifugal instability of nanofluid flows. The book also outlines criteria for both types of instability. Applications of liquids with nanoparticles in quenching processes were analyzed.

The present book consists of nine chapters. The book focuses on convective heat and mass transfer in boundary layers, during steady-state and unsteady condensation and boiling of nanofluids, as well as on the issues of instability at boiling and centrifugal instability. Chapters 1 and 2 are devoted to the physical foundations and methods of mathematical modeling of nanofluids. Chapters 3–8 present the results of solving various problems related to the flows of nanofluids, whereas each of them begins with an introduction, which provides a brief overview of the work of various researchers and formulates the purpose and objectives of this chapter.

Chapter 1 outlines physical foundations and mathematical models of transport processes in nanofluids. This chapter briefly shows that the addition of high thermal conductivity nanoparticles to conventional fluids intensifies heat transfer in them. Chapter 1 shows how mathematical models can consider the effect of nanoparticles on thermal conductivity, dynamic viscosity, heat capacity, and density of nanofluids. Later in this chapter, the thermophoresis and Brownian diffusion models are considered as the main physical mechanisms that influence the transport processes in nanofluids. In conclusion, a mathematical model of fluid flow and heat and mass transfer in nanofluids, which is used throughout this book, is presented in detail.

In Chap. 2, the foundations of two analytical mathematical methods used in mathematical modeling in this monograph are outlined: Lie group theory (symmetry analysis) and perturbation method.

Chapter 3 is devoted to an analysis of boundary layer flows (parabolic flows) of nanofluids as homogeneous single-phase fluids (i.e., flows without phase transition). This chapter outlines symmetry and self-similar forms of equations for different kinds of boundary layers on a flat surface. After that, self-similar solutions of the problems of flow and heat and mass transfer are presented for the laminar and turbulent boundary layers in the case of forced flow over a horizontal surface. In the last section, the problem of orthogonal impingement of a flow onto a flat wall is considered.

In Chap. 4, results are presented for the problems of film condensation of (a) a stationary vapor with nanoparticles on a vertical surface and (b) a moving vapor that moves in the same direction with a film of condensate on a horizontal surface. The obtained analytical solutions are an extension and further elaboration of the classical Nusselt solution.

Chapter 5 describes the results for laminar flow, heat, and mass transfer in a vapor film on a vertical surface with developed steady-state film boiling of a stationary nanofluid. The model used here is a further development of the works of Bromley, Ellion, and Koh for ordinary fluids. As a result, an analytical solution to the problem was obtained. In conclusion, symmetry analysis was performed, and self-similar forms of transport equations were obtained, which were solved numerically. This solution was analyzed in comparison with the analytical solution.

In Chap. 6, results for the problem of instantaneous transition to film boiling in ordinary fluids and nanofluids on a vertical surface for the case of a sudden heat supply are outlined and analyzed. Here two approaches are used: the Laplace transform and the symmetry method. Self-similar equations obtained using the Lie group transformations were solved numerically. Validations were performed using the CFD methodology and experimental measurements of surface heat transfer of a metal probe, very quickly immersed in a cooling nanofluid.

Chapter 7 focuses on the study stability of the flow in a vapor film formed at boiling of nanofluids on a vertical surface, whereas the vapor flow in the film is directed vertically upward. The modeling was performed using the method of linear perturbations in two-dimensional and three-dimensional approximations. Experimental validation was carried out on the basis of data on the effect of nanoparticles on the formation and destruction of a vapor film formed during the boiling of nanofluid on the surface of a metal probe during unsteady cooling.

In Chap. 8, original results are presented of modeling using the perturbation method of the stability criteria in the case of instability during boiling, as well as centrifugal instability in the Dean and Taylor–Couette flows of nanofluids with account for the radial temperature gradient.

Chapter 9 presents summary and overall conclusions to the book.

The results that form the basis of this book were obtained during the last decade of our joint creative collaboration. We want to thank all of our colleagues with whom we have collaborated during this time, for their contributions, helpful advice, and friendly discussions.

We are very grateful to our families for their invaluable continued support and understanding in the preparation of this book.

Kyiv, Ukraine — Andriy A. Avramenko
Gummersbach, Germany — Igor V. Shevchuk

Contents

Nomenclature

$\mathrm{Ar} = \frac{g\delta^3}{\mu^2}\rho\Delta\rho$	Archimedes number (–)
$c_w = \frac{2\tau_w}{\rho_\infty U_\infty^2}$	Skin friction coefficient (–)
$c_{w0} = \frac{2\tau_{w0}}{\rho_\infty U_\infty^2}$	Skin friction coefficient for a pure base fluid (–)
c	Isobaric specific heat of a fluid (J/(kg K))
c_υ	Isobaric specific heat of pure vapor without nanoparticles (J/(kg K))
c_p	Specific heat of the nanoparticles (J/(kg K))
D_B	Brownian diffusion coefficient (m^2/s)
D_{Bt}	Turbulent Brownian diffusion coefficient (m^2/s)
D_{Beff}	Effective Brownian diffusion coefficient (a sum of molecular and turbulent values) (m^2/s)
D_T	Thermophoretic diffusion coefficient (m^2/s)
D_{Bt}	Turbulent thermophoretic diffusion coefficient (m^2/s)
D_{Teff}	Effective thermophoretic diffusion coefficient (a sum of molecular and turbulent values) (m^2/s)
$\overline{D} = \frac{D_T}{D_B}$	Ratio of the thermophoretic diffusion coefficient to the Brownian diffusion coefficient (–)
$D = \frac{\Delta T}{T_\infty}\frac{D_T}{D_B}$	Parameter (–)
d_p	Particle diameter (m)
$\mathrm{Ec} = \frac{U_\infty^2}{c_f \Delta T}$	Eckert number (–)
$\mathrm{Fo} = \frac{\alpha t}{R^2}$	Fourier number based on the thermal diffusivity (–)
$\mathrm{Fo}^* = \frac{t\nu}{\delta^2}$	Fourier number based on kinematic viscosity (–)
$\mathbf{g}$	Acceleration of gravity (m/s^2)
G	Mass flowrate per unit length (kg/(m·s))
G_l	Mass flow rate through the film per unit of surface area (kg/(m^2·s))
$\mathrm{Ga} = \frac{gx^3\rho_f^2}{\mu^2}$	Galilei number (–)
$h = cT$	Specific enthalpy (J/kg)
h_w	Specific enthalpy of at the wall (J/kg)

h_∞	Specific enthalpy of the outer flow, i.e., the flow outside of the boundary layer (J/kg)
h_T	Heat transfer coefficient (W/(m^2 K))
$\boldsymbol{j_p}$	Mass flux of nanoparticles (kg/m^2·s)
$\mathrm{Ja}_h = \frac{h_\infty}{L_\upsilon}$	Jacoby number (–)
k	Thermal conductivity of the nanofluid (W/(m K))
k_f	Thermal conductivity of the pure base fluid (W/(m K))
k_p	Thermal conductivity of the nanoparticles (W/(m K))
k_t	Turbulent thermal conductivity of the nanofluid (W/(m K))
k_{eff}	Effective thermal conductivity of the nanofluid (a sum of molecular and turbulent values) (W/(m K))
$K_{pf} = k_p/k_f$	Normalized thermal conductivity of nanoparticles relative to that of the nanofluid (–)
$K_{p\upsilon} = k_p/k_\upsilon$	Normalized thermal conductivity of nanoparticles; relative to that of pure vapor (–)
$k_B = 1.38065\cdot10^{-23}$	Boltzmann constant (J K^{-1})
L_c	Characteristic length (m)
L_υ	Latent heat of vaporization (J kg^{-1})
$\mathrm{Le} = \alpha_f/D_B$	Lewis number (–)
$\mathrm{Le} = \frac{k_f}{\rho_f c_f D_B}\frac{\rho_f c_f}{\rho_p c_p}$	Modified Lewis number of a pure base fluid based on the Brownian diffusion coefficient (–)
$\mathrm{Le} = \frac{\mathrm{Sc}}{\mathrm{Pr}}\frac{\rho_\upsilon c_\upsilon}{\rho_p c_p}$	Modified Lewis number of a pure vapor based on the Brownian diffusion coefficient (–)
$\mathrm{Nu} = h_T L_c/k$	Nusselt number for ordinary fluids (–)
$\mathrm{Nu} = \frac{h_T x}{k_f}$	Local Nusselt number for nanofluids (–)
$\mathrm{Nu} = \frac{h_T \delta 0}{k_f}$	Nusselt number for nanofluids based on the condensate film thickness (–)
$\mathrm{Pr} = \frac{\mu_f c_f}{k_f} = \nu_f/a_f$	Prandtl number of a pure base nanofluid (–)
$Pr = \frac{\mu_\upsilon c_\upsilon}{k_\upsilon} = \nu_v/a_v$	Prandtl number of a pure vapor (–)
p	Static pressure (Pa)
Q	Heat flux (W)
q	Heat flux per unit area (W/m^2)
$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$	Wall heat flux per unit area (W/m^2)
q_{cr}	Critical heat flux per unit area (W/m^2)
$R_{pf} = \rho_p/\rho_f$	Ratio of the densities of nanoparticles and pure base fluid (–)
$R_{p\upsilon} = \frac{\rho_p}{\rho_\upsilon}$	Ratio of the densities of nanoparticles and pure vapor (–)
$\mathrm{Re} = VL_c/\nu$	Reynolds number (–)
$\mathrm{Re}_x = \frac{\rho_f U_\infty x}{\mu_f}$	Local Reynolds number for nanofluids (–)
$\mathrm{Re}_\delta = \frac{U_\infty \delta \rho}{\mu}$	Characteristic Reynolds number (–)
$\mathrm{Re}_\tau = \frac{\sqrt{\tau_\infty/\rho}\,\delta}{\mu}$	Characteristic Reynolds number (–)

$\mathrm{Sc} = \frac{\mu_f}{\rho_f D_B} = \frac{\nu_f}{D_B}$	Particle Schmidt number of a nanofluid based on the Brownian diffusion coefficient (–)
$\mathrm{Sc}_T = \frac{\mu_f}{\rho_f D_T} = \nu_f/D_T$	Particle Schmidt number of a nanofluid based on the thermophoretic diffusion coefficient (–)
$\mathrm{Sc} = \frac{\mu_v}{\rho_v D_B} = \nu_v/D_B$	Particle Schmidt number of a pure vapor based on the Brownian diffusion coefficient (–)
$\mathrm{St}_0 = \frac{h_{T0}}{c_\infty \rho_\infty U_\infty}$	Stanton number
t	Time (s)
$\tilde{t} = \frac{t\nu}{\delta^2}$	Dimensionless time (–)
$t^* = \frac{\tilde{t}}{\mathrm{Pr}}$	Dimensionless time (–)
T	Local temperature (K)
T_w	Wall temperature (K)
T_∞	Ambient temperature, i.e., temperature outside of the boundary layer (K)
T_{sat}	Saturation temperature at a given pressure (K)
V	Characteristic velocity (m/s)
$\mathbf{v}$	Fluid velocity vector (m/s)
$\nabla\mathbf{v}$	Tensor gradient of velocity ($s^{-1}m^{-1}$)
$(\nabla\mathbf{v})^T$	Conjugate tensor gradient of velocity ($s^{-1}m^{-1}$)
v	Wall-normal velocity component (y-component) (m/s)
v, u, w	Radial, azimuthal, and axial components of the flow velocity in cylindrical polar coordinates r, φ, z (m/s)
u, v, w	Velocity components in Cartesian coordinates (m/s)
u	Streamwise velocity component (x-component) (m/s)
U_∞	Outer flow velocity, i.e., flow velocity outside of the boundary layer (m/s)
x, y, z	Cartesian coordinates (m)
α	Volumetric thermal expansion coefficient of the nanofluid (K^{-1})
α	Thermal diffusivity of the nanofluid (m^2/s)
δ	Thickness of a momentum boundary layer, condensate film or vapor film (m)
$\Delta T = T_w - T_\infty$	Temperature difference in the boundary layer (K)
$\Delta T = T_w - T_{sat}$	Wall superheat during boiling (K)
$\eta = \frac{y}{\delta}$	Dimensionless coordinate in the condensate or vapor film (–)
$\Theta = \frac{T - T_\infty}{T_w - T_\infty}$	Non-dimensional temperature (–)
$\Theta = \frac{T - T_\infty}{T_w - T_\infty} \Theta = \frac{T_w - T}{T_w - T_\infty}$	Non-dimensional temperature in the unsteady heat transfer problems (–)
μ	Dynamic viscosity of the nanofluid (Pa s)
μ_f	Dynamic viscosity of the pure base fluid (Pa s)
μ_t	Turbulent viscosity of the nanofluid (Pa s)
μ_{eff}	Effective dynamic viscosity of the nanofluid (a sum of molecular and turbulent values) (Pa s)

ν	Kinematic viscosity of the nanofluid (m^2/s)
ρ	Density of the nanofluid (kg/m^3)
ρ_f	Density of the pure fluid (kg/m^3)
ρ_p	Density of the nanoparticles (kg/m^3)
ρ_∞	Density of the nanofluid outside of the boundary layer (kg/m^3)
ρ_v	Density of pure vapor (kg/m^3)
$\bar{\rho} = \rho_\infty / \rho_f$	Non-dimensional density outside of the boundary layer (–)
τ	Shear stress (Pa)
$\tau_w = \mu_w \left(\frac{\partial u}{\partial y}\right)_{y=0}$	Shear stress on the wall (Pa)
φ	Volume fraction (concentration) of the nanoparticles (–)
φ_w	Volume fraction (concentration) of the nanoparticles, on the wall (–)
φ_∞	Volume fraction (concentration) of the nanoparticles, outside of the boundary layer (–)

Subscripts

av	Average value
f	Properties of a pure base fluid
p	Particles (i.e., nanoparticles)
t	Turbulent parameters
υ	Properties of a pure vapor
w	Wall value (at $y = 0$)
0	Standard conditions: a pure base fluid
∞	Outer flow (i.e., outside of the boundary layer)

Mathematical Symbols

Boldface	Vector values
—	Operator Nabla
$\nabla^2 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}\right)$	Operator Nabla quadrat
δ	Unit tensor (Kronecker tensor)

Acronyms

CFD	Computational fluid dynamics
DPHC	Direct problems of heat conduction
IPHC	Inverse problems of heat conduction
MATHLAB	Programming and numeric computing platform
VOF	Volume of liquid