APPLICATION OF SERIES SEGMENTS BY ORTHOGONAL POLYNOMIALS FOR STATISTICAL ESTIMATION OF PROBABILITY DENSITY

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Objective. When solving problems of noise diagnostics of elements of heat-and-power engineering objects, it is necessary to know the probability densities of both the most diagnostic signals and their parameter estimators. Since the exact expressions of probability densities are usually unknown, for their finding, in particular, representations in the form of series segment by classical orthogonal polynomials are used. The purpose of this paper is to study the probabilistic characteristics of the probability density estimating in the form of a series segment by orthogonal polynomials of Hermite, Laguerre and Jacobi (Legendre and Chebyshev of the first and second kind).

Results. The probability density estimator is based on a sample of random variables, which are the instantaneous values of the noise diagnostic signals. It is shown that, in the general case, the probability density estimator o in the form of a series segment has a bias, which is a systematic error and does not depend on the sample size. The use of series segments of orthogonal polynomials up to the sixth order, as well as generalized Feier and Vallée-Poussin sums on them, allows to obtain integral approximation errors of classical theoretical probability densities that do not exceed 0.1.

Formulas for the variance and correlation function for probability density estimator were obtained, which showed that the estimator is consistent, and allowed us to find the relative estimation error and the contour probability density estimators based on the asymptotically normal distribution law of the expansion coefficients, which are determined by moments of the diagnostic signal.

In the statistical estimation of the probability density by computer simulation, it was found that the integral root-mean-square errors in estimating the probability densities may differ significantly from the theoretical values; however, with a sample size of 10^6, they do not exceed 0.003. It is also confirmed that the possible implementations of the estimates of the probability density lie within the limits that are determined by the three standard deviations of the estimator.

Conclusions. The use of series segments by classical orthogonal polynomials allows approximation of probability densities that differ from zero on the infinite and semi-infinite axes of the real numbers, as well as on the interval of real numbers with errors that are acceptable for solving diagnostic problems.